

# The Convergence Criterion in Measuring Building R-Values

S.N. Flanders

## ABSTRACT

*ASTM C 1155, Standard Practice for Determining Thermal Resistance of Building Envelope Components from In-Situ Data, requires that a calculation of R-value converge on a steady value within criterion bounds. Some field experience suggests that this criterion is often easily met. Some theoretical studies suggest that the criterion may be blind to certain temperature-change conditions.*

*In this paper, long-term field data are converted into synthetic pure wave forms and employed in a computational model. The convergence criterion is tested for several delay periods to determine the worst case for that set of frequencies. The investigation highlighted the effect on R-value calculations of low-frequency temperature inputs, representing periodicities of up to 500 hours. The mathematical simulations allow discrimination between conditions that cause lack of convergence and those that represent a long-term change in R-value.*

*While the convergence strategy suggested in C 1155 is adequate, a more sophisticated algorithm would increase the reliability of the test. The automated data collection process would include periodic Fourier transforms of the hourly data that have been collected, perform a power spectrum analysis, and select the most stringent delay period for performing the convergence test.*

## INTRODUCTION

### Determination of In Situ Thermal Resistance

Determination of thermal resistance, or R-value, of in-place walls, roofs, floors, or other elements is useful (1) to ensure that new construction complies with required insulation levels and (2) to assess the current insulation performance of existing buildings in order to determine the value of adding insulation. The American Society for Testing and Materials (ASTM) has published several standards to make this possible. They encompass the collection of temperature and heat flux data and calculation of thermal resistance.

**Collection of Data** ASTM C 1046-91 "details a technique for using heat flux and temperature transducers in measurements of the dynamic or steady-state thermal behavior of opaque building components" (ASTM 1991). The standard references ASTM C 1130 for sensor calibration (ASTM 1990) and incorporates the use of infrared

thermography. Briefly, heat flux is measured on the inside surface of the building component and temperature is measured on the inside and outside surfaces. All data are recorded periodically.

**Calculation of Thermal Resistance** ASTM C 1155-90 "details how to use data obtained from in-situ measurement of temperatures and heat fluxes on building envelopes to compute thermal resistance" (ASTM 1990). The technique entails the approximation of R-value ( $R$ ) as follows:

$$R(t) = \frac{\sum_{t=0} [T(t)_{in} - T(t)_{out}]}{\sum_{t=0} q(t)_{in}} \quad (1)$$

where

$t$  is time and  $T$  and  $q$  are the temperature and heat flux at a surface.

Other techniques use estimates of the heat capacity of the building element (Anderson 1986), statistical optimization of time constants in a thermal model of the element (Roulet et al. 1986), or parametric estimation of the thermal properties of the component layers (Beck et al. 1991). No matter what the method of calculation, there is a minimum duration of measurement that results in a valid calculation of R-value.

**Sufficient Data Criterion** If one obtains temperature and heat flux data for an insufficient duration, the calculation in Equation 1 may be significantly biased by not obtaining the temperature history prior to the onset of heat transfer measurement or the post-measurement heat flux that resulted from the temperature history prior to the cessation of measurement. ASTM C 1155 suggests comparing  $R(t)$  with  $R(t-n)$  for some suitable time period  $n$ :

$$CR_n = \frac{R(t) - R(t-n)}{R(t)} \quad (2)$$

When  $|CR|$  remains less than 10% for a period of more than  $3n$ , then the convergence criterion has been satisfied and  $R$  in Equation 1 may be used.

The problem with choosing this criterion is deciding which value of  $n$  to use. ASTM C 1155 suggests using the value that results in the most severe test for satisfying the criterion. It recommends starting at six hours and then trying other variations,  $1 < n < 12$  hours. The standard then allows one to estimate the variance of  $R$  with at least three such independent measurements and calculations.

The purpose of this paper is to determine an effective means to assess when enough data have been collected for a sufficiently accurate calculation of R-value. A previous paper (Flanders 1991) used a model to address this question but stopped short of incorporating field data in the analysis of realistic temperature conditions.

**Determining the Presence of Changing R-Value** A problem occurs when using in-situ data to determine whether the convergence criterion is adequate. One cannot discriminate between a change in R-value with temperature or moisture content and an insufficient duration of calculation to calculate a stable R-value. This ambiguity does not arise when calculating R-value in a simulation that assumes constant material thermal properties. Therefore, field data and simulated measurements made it possible to examine the convergence criterion as discussed below.

## PROCEDURE

The procedure, in brief, was to obtain in-situ heat flux and temperature data according to ASTM C 1046 and calculate R-value according to ASTM C 1155. A Fourier analysis determined the dominant frequencies in the in-situ data and made possible an artificial data set suitable for use in a simulation. The simulation made possible a comparison of different convergence strategies with the true error of  $R(t)$ , when compared with the assumed R-value in the simulation.

### Three Sources of In-Situ Data

A contractor measured R-values according to ASTM C 1046 and C 1155 for three buildings and reported the data in print and on floppy disks (Desjarlais 1991; Kunz 1992). The buildings were as follows:

**Metal Building in North Dakota** An Army Corps of Engineers maintenance building (MB)\* at Garrison Dam in Riverdale, North Dakota, was monitored for 487 hours, commencing December 11, 1990. The building walls were 50 mm (2 in.) thick insulated metal panels. The insulation type was never verified but was assumed to be fiberglass. Instrumentation was placed on insulated wall panels as follows: a grouping of three on a south wall and a grouping of three on a north wall. Additionally, a grouping of three was placed on a north-facing concrete masonry unit (CMU) half-wall that was covered with insulated panel on the outside.

**Metal Building in New York State** An Army vehicle maintenance (VM) shop at Fort Drum, New York, was monitored for 330 hours, commencing December 4, 1991. Again, the building walls were 50 mm (2 in.) thick insulated metal panels. The insulation type was never verified

\*This and other abbreviations are keyed to other references to buildings in this paper, especially in naming sensor sites.

but was assumed to be fiberglass. Instrumentation was placed at eight sites on insulated wall panels, vertically spaced about 1 m (40 in.) apart. Two sensor sites were on a block foundation wall.

**Attic in New York State** An Army headquarters (HQ) building at Fort Drum, New York, was monitored for 240 hours, commencing December 20, 1991. The masonry block walls with a brick veneer exterior contained a 63.5-mm (2.5-in.) layer of "rigid insulation," according to the as-built plans. The attic space contained nominal 200-mm (8-in.) fiberglass batts loosely laid on the gypsum board ceiling. Six sensor sites were on the wall at vertical spacings of between 0.4 and 0.6 m (16 and 24 in.), and four were on the insulated ceiling at 0.6-m (24-in.) spacings.

**Five Sensor Sites Selected** This paper focuses on five sensor sites that represent a variety of conditions. They are summarized in Table 1.

MB1, MB2, and VM1 represent similar construction with different temperature signals. The sensor site for VM1 was on a seam between two panels. MB1 and MB3 represent different constructions with similar orientation. HQ1 has nominally four times the thermal resistance of the other cases. Discussion of the HQ masonry veneer wall with its air space is beyond the scope of this paper.

### Analysis of Dominant Frequencies

The procedure for determining the dominant frequencies in the field data was to perform a Fourier analysis of the temperature data, compile a power value for each frequency, choose the dominant frequencies with which to reconstruct the data, and confirm their correspondence to the original data.

**Perform Fourier Analysis** A discrete Fourier transform (DFT) of a time series of data produces real and imaginary components of a series expansion, using sines and cosines, that can closely approximate the original data series (Stearns and David 1988). For efficiency, the DFT was applied to the deviation of each datum from the mean of the data.

**Choose Dominant Frequencies** The sum of the squares of each real and imaginary component represents

TABLE 1  
Summary of the Five Sensor Sites  
Discussed in this Paper

Reference	Orientation	Construction of Component
MB1	North	50-mm (2-in.) insulated metal panel
MB2	South	50-mm (2-in.) insulated metal panel
MB3	North	CMU with insulated metal panel
VM1	South	50-mm (2-in.) insulated metal panel
HQ1	Up	Ceiling with 254-mm (8-in.) batts

the power of each frequency. Choice of the frequencies with the four highest powers and substitution of their real and imaginary components into the inverse DFT produces a smoothed version of the original data set (Figures 1 through 4) in sufficient detail to test convergence.

**Confirm Correspondence to Data** The data were reconstructed for each of the five sensor sites by computing the inverse DFT of the four chosen frequencies, adding the mean values of the original data to the result, and comparing the resulting curve with the original data, similar to Figures 1 through 4.

### Simulation of Thermal Response

The reconstructed data were compatible with a computational model, using the admittance method, which previously assessed convergence in R-value calculations (Flanders 1991). This model permitted calculation of thermal resistance and convergence for comparison with

given assumptions. The model predicts approximate heat flows from a specified temperature series (Reddy and Krishnamoorthy 1989) and therefore was useful to test strategies relative to each other, rather than in absolute comparison with the field data.

**Employ Frequency and Material Components in the Simulation** The model calculates admittances based on sinusoidally changing inside and outside temperatures and on the sequence of materials layered within the component. The DFT provided the dominant four frequencies, expressed in terms of sines, cosines, and their amplitudes. Field inspection of as-built drawings and the actual building components provided assumed thicknesses and material specifications of the layers.

**Calculate Thermal Resistance and Convergence** The admittance calculation provides a mechanism for calculating a simulated temperature and heat flux time series. With this, one may test a variety of convergence criteria or other tests against the R-value that is defined within the model.

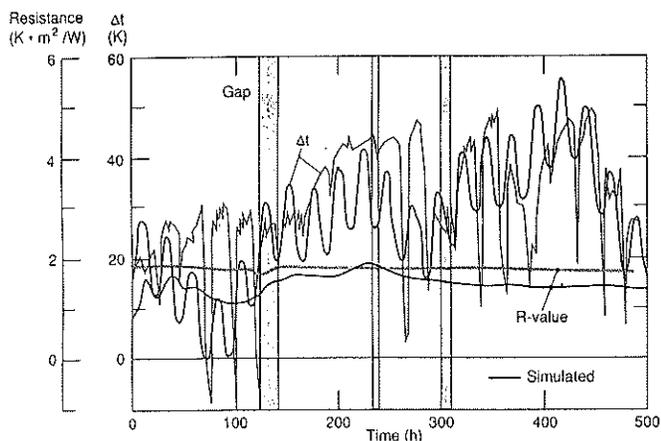


Figure 1 Measured and modeled R-value ( $10 \text{ K}\cdot\text{m}^2/\text{W}$ ) and  $\Delta T(\text{K})$  for sensor site MB2.

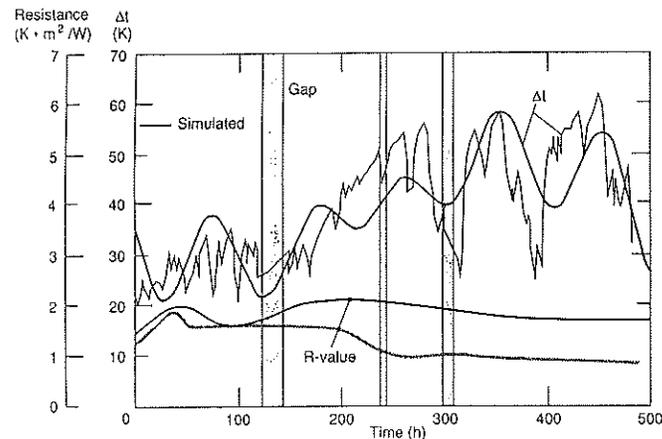


Figure 2 Measured and modeled R-value ( $10 \text{ K}\cdot\text{m}^2/\text{W}$ ) and  $\Delta T(\text{K})$  for sensor site MB3.

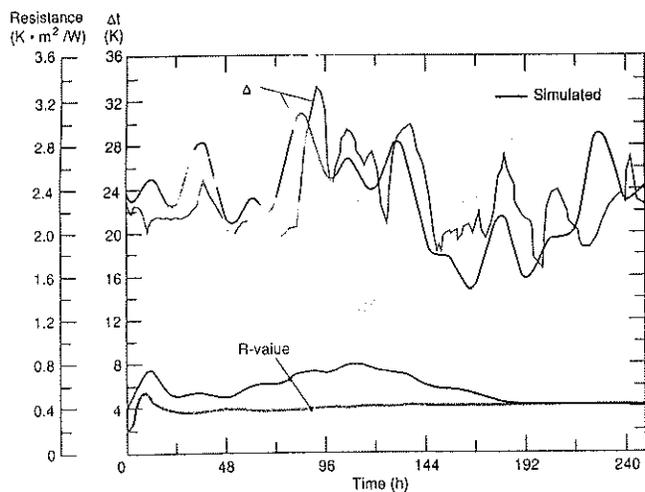


Figure 3 Measured and modeled R-value ( $\text{K}\cdot\text{m}^2/\text{W}$ ) and  $\Delta T(\text{K})$  for sensor site HQ1.

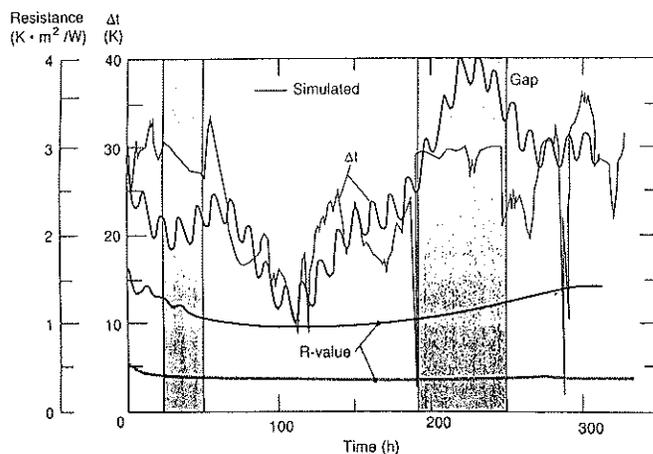


Figure 4 Measured and modeled R-value ( $10 \text{ K}\cdot\text{m}^2/\text{W}$ ) and  $\Delta T(\text{K})$  for sensor site VM1.

## Comparison of Simulations with Measurements

Some of the field data records had gaps. Interpolations of temperatures and heat fluxes at these gaps created a long record in each case to compare with the model. The Fourier series obtained for modeling used the continuous, interpolated data sets.

**Modeled vs. Measured R-Values** One may compare one's assumptions, made by looking at the component and the drawings, with the field data. Discrepancies between the two may reflect on those assumptions, on the performance of the building component, or on the validity of the measurement.

**Modeled vs. Measured Convergence of R-Values** One may next compare how the same convergence rule behaved in the model environment with how it behaved in the field measurement. A discrepancy may reflect on the model's validity.

**Convergence vs. Modeled Error** One may also compare convergence rules in the model with the actual error. A convergence rule in the modeled environment that ensures that the actual error is less than the convergence value of the moment will ensure that the measurement duration is sufficient.

## RESULTS

### R-Value and Convergence from In-Situ Data

**Metal Building in North Dakota** The average measured R-value at equivalent locations in the metal panels (MB1, MB2, and a replicate) was  $1.6 \text{ K}\cdot\text{m}^2/\text{W}$  ( $9.2^\circ\text{F}\cdot\text{h}\cdot\text{ft}^2/\text{Btu}$ ) with a standard deviation of 8% from location to location. The R-value for the metal-panel-covered masonry wall at MB3 was  $1.0 \text{ K}\cdot\text{m}^2/\text{W}$  ( $5.7^\circ\text{F}\cdot\text{h}\cdot\text{ft}^2/\text{Btu}$ ). This sensor site was at the top of the half-wall and least coupled with the foundation of such sites. The test lasted 487 hours. Three gaps in the data occurred as a result of equipment or power failure: a 26-hour gap at  $t=121$  hours, a 2-hour gap at  $t=239$  hours, and a 13-hour gap at  $t=300$  hours.

The average six-hour convergences for M1, MB2, and MB3 were well below 1%. The average variance in measured R-value for blocks of time that met the six-hour convergence criterion was 2% of the average measured R-value at MB1, 8% at MB2, and 43% at MB3, the masonry location.

**Metal Building in New York State** The R-value for the metal panel at VMI was  $0.44 \text{ K}\cdot\text{m}^2/\text{W}$  ( $2.5^\circ\text{F}\cdot\text{h}\cdot\text{ft}^2/\text{Btu}$ ). This sensor site was on a seam between panels. Five sites in the middle of such panels had an average R-value of  $1.1 \text{ K}\cdot\text{m}^2/\text{W}$  ( $6.0^\circ\text{F}\cdot\text{h}\cdot\text{ft}^2/\text{Btu}$ ). The test lasted 330 hours. Two gaps in the data occurred as a result of equipment or power failure: a 30-hour gap at  $t=22$  hours and a 59-hour gap at  $t=190$  hours.

The average six-hour convergence at VMI was 0.2%. The average variance in measured R-value for blocks of time that met the six-hour convergence criterion was 17% of the average measured value.

**Attic in New York State** The R-value for the attic insulation system at HQ1 was  $4.4 \text{ K}\cdot\text{m}^2/\text{W}$  ( $25^\circ\text{F}\cdot\text{h}\cdot\text{ft}^2/\text{Btu}$ ). This sensor site was on an insulated ceiling. The cold-side temperature sensors were in an attic space. The test lasted 240 hours. There were no gaps in this data set.

The average six-hour convergence at HQ1 was 0.02%. The average variance in measured R-value for blocks of time that met the six-hour convergence criterion was 9.8% of the average measured value.

### Dominant Frequencies Encountered

Table 2 gives a summary of the dominant periodicities (1/frequency) encountered, ranked according to their power, as calculated from the DFT. In each case, the first time period reflects the length of the measurement. A longer measurement period might have identified the most powerful frequencies to have even longer periodicities. Note that the expected 24-hour diurnal cycle was often not a dominant periodicity compared with other temperature periodicities. Addition of two more periodicities did not appreciably alter the simulated  $\Delta T$  data.

**Confirm Correspondence to Data** Figures 1 through 4 illustrate how inverse DFT data provided an approximate reconstruction of the original data. The largest discrepancy between the actual and the simulated  $\Delta T$  is for VM1, where the gaps in the data were largest compared to the data set.

### Comparison of Simulations with Measurements

A comparison of the long-term interpolated R-values with those calculated in the true data sets revealed that the interpolations did not significantly distort the measured R-value calculations.

**Modeled vs. Measured R-Values** Figures 1 through 4 also illustrate the comparison between the measured

TABLE 2  
Summary of the Dominant Periodicities Encountered

Site	Periodicities (h)			
	First	Second	Third	Fourth
MB1	486	243	97	122
MB2	486	243	24	97
MB3	486	243	122	97
VM1	330	165	83	30
HQ1	238	119	48	24

(using data interpolated at gaps) and the modeled R-values. Sites MB1 and MB2 (Figure 1) had measured R-values that were 25% and 6% better than the assumed calculated value. For site MB3 (Figure 2), the measured R-value was 60% of the calculated value. The sites on this masonry half-wall were progressively more coupled with the ground the closer they were to the slab-on-grade.

The measured R-value in the attic for sensor site HQ1 was virtually identical to the assumed value for that location (Figure 3). However, the measured R-value for VM1 at the seam of a metal panel (Figure 4) was only 30% of what one might have calculated for that panel. The average measured R-value for five sites that were in the centers of such panels was only 75% of what might be expected for a 50-mm (2-in.) glass-fiber-insulated panel.

**Modeled vs. Measured Convergence of R-Values** The field results confirm the assumption (Reddy and Krishnamoorthy 1989) that the model only approximates the correct heat flux. Figures 5 and 6 illustrate differences in how rapidly the measurement and the model achieve convergence. Figure 5 depicts CR (Equation 2), convergences of measured R-value, for  $n = 12, 48,$  and 120 hours in

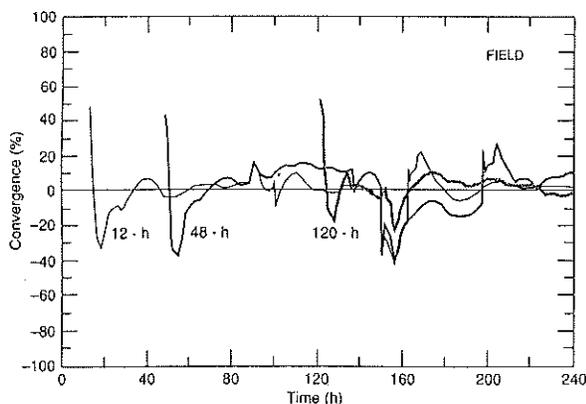


Figure 5 Measured 12-, 48-, and 120-hour convergences for sensor site HQ1.

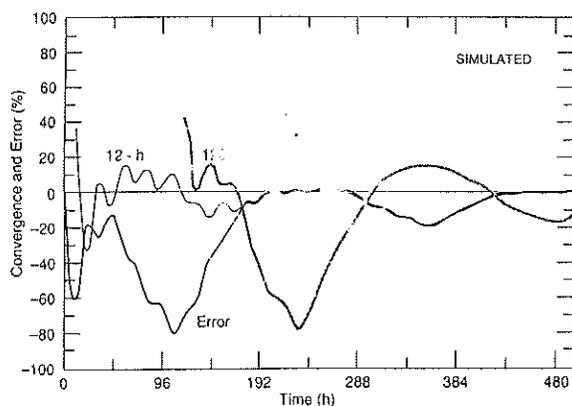


Figure 6 Modeled 12- and 120-hour convergences and calculation error for sensor site HQ1.

Equation 2 for HQ1, an example with continuous data and good correspondence between the field data and the model. Figure 6 represents twice the duration of Figure 5 for the modeled cases of  $n = 12$  and 120 hours and for the modeled error. The modeled error is the percentage difference between the assumed R-value and that calculated in the model by Equation 1. Consequently, the model is pessimistic about convergence compared with the field measurements.

**Convergence vs. Modeled Error** The essential concern of this paper is how well a chosen strategy predicts the error of the calculation. With the convergence strategy (Equation 2), one should choose  $n$  such that CR is likely to be less than the absolute value of the modeled error in the calculation of R (Equation 1). The simulation in Figure 6 shows that, for HQ1, a choice of  $n = 120$  hours (half the longest periodicity) is a very conservative test for error. The CR curve reaches the same absolute maximum as the error curve at a significantly later time.

**Detecting a Change in Measured R-Value** Site MB3 (Figure 3) underwent a change of R-value after about 200 hours from 1.6 to 0.90 K·m<sup>2</sup>/W (9.1 to 5.1°F·h·ft<sup>2</sup>/Btu). Examination of the model simulation reveals that the agreement between model and measured R-value is good for the first 100 hours and that the model never varies by more than 25%, whereas the measured values drop by 44%. This suggests that lateral flow to the increasingly cold slab-on-grade is responsible for the greater drop in the measured R-value.

## DISCUSSION

### New Minimum n Indicated

The application of ASTM C 1155 accepted R-value measurements at all sites except MB3. Although in no case did the six-hour choice of  $n$  fail to select an R-value calculation that was within 10% of the long-term value, a 12-hour  $n$  may be a more appropriate minimum value. The use of a 24-hour-period sine-wave temperature in the model demonstrated that the minimum appropriate value for  $n$  is 12 hours to address the maximum error for that periodicity. Hence, for other periodicities, half the values would be preferred values for  $n$ . Some periodicities that approximately correspond to those encountered would be 240, 120, 96, 48, and 24 hours.

### Application of $n = 12$ hours

For the lightly insulated, light-mass walls—MB1, MB2, and VM1—measured Rs were within 10% of the final value within two hours and did not stray from those bounds thereafter. The highly insulated (HQ1) and massive (MB3) cases offer less clear lessons. HQ1 remained within 10% of the ultimate value after 65 hours. Using a choice of  $n = 12$

hours,  $CR$  fell below 10% at 28 hours. According to ASTM C 1155, one would end the measurement after 64 hours, that is, 28 + 36 hours. MB3 remained within 10% of the 200-hour value after 48 hours. Using a choice of  $n = 12$  hours,  $CR$  fell below 10% at 35 hours. According to ASTM C 1155, one would end the measurement after 71 hours, that is, 35 + 36 hours. After 200 hours, lateral heat flow to the slab-on-grade may have changed the ultimate  $R$ -value.

### **Model Useful for Determining Sampling Strategy**

Although the model's calculation of heat flux is approximate, it correctly computes  $R$ , according to Equation 1. A result of this is that the model overpredicts the magnitude of convergence. One therefore must use it for comparisons within the modeling environment, rather than for comparisons between measurements and models. Judicious use of assumed amplitudes for 24-, 48-, 96-, and 240-hour cycles makes it useful for deciding in advance how long one may expect to measure.

### **CONCLUSIONS**

#### **Current Standard is Adequate but Improvable**

The current standard is adequate, but the recommended minimum value for  $n$  should be changed from six hours to twelve hours. An automated application of ASTM C 1155—requiring  $CR$  to remain less than 10%—may be useful.

#### **Real-Time Fourier Analysis Recommended**

A sophisticated on-site measurement program could perform Fourier transforms periodically to determine whether some long periodicities are lurking in the data. A look-up table that compares the periodicities with criteria based on the evolving  $R$  and the mass assumed by inspection would determine whether enough data have been obtained.

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